Admin: • Drop date is Tuesday Sept 19. Diagnostic < 40%.

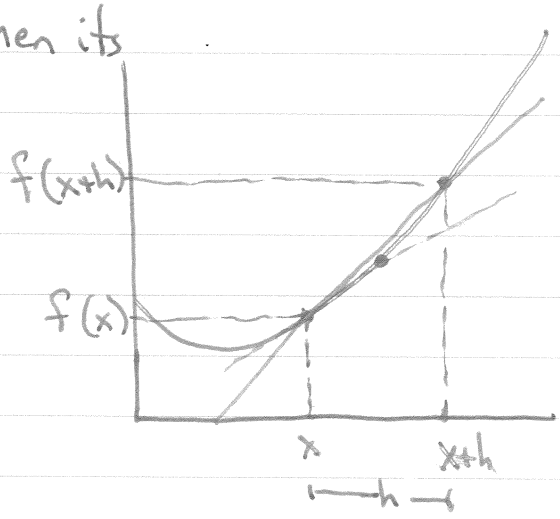
• Quiz on Friday Sept 22. Material up through today.

Last: Limits & the Derivative

Today: • Continuity of functions
• Practice!

Last time: If $f(x)$ is a function, then its derivative is the function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



For any point x_0, the value $f'(x_0)$ is equal to the slope of the tangent line to $y = f(x)$ at $x = x_0$.

Ex. $f(x) = Cx^2$

$$f'(x) = \lim_{h \to 0} \frac{C(x+h)^2 - Cx^2}{h} = \lim_{h \to 0} \frac{2Cxh + Ch^2}{h} = \lim_{h \to 0} (2Cx + Ch)$$

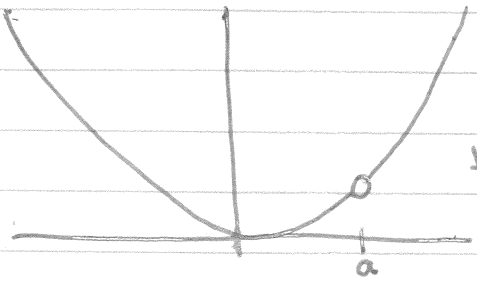
$$= 2Cx$$

Questions: • Does the derivative always exist?
• What qualitative things about $f(x)$ can we get from its derivative?
• How to compute limits? When does Plugging in work?

$x^2(x-a)$

1. $f(x) = \dfrac{x^3 - ax^2}{x-a} = x^2$

$\lim_{x \to a} f(x) = a^2$



The limit is talking about points close to a. (NOT $x=a$)

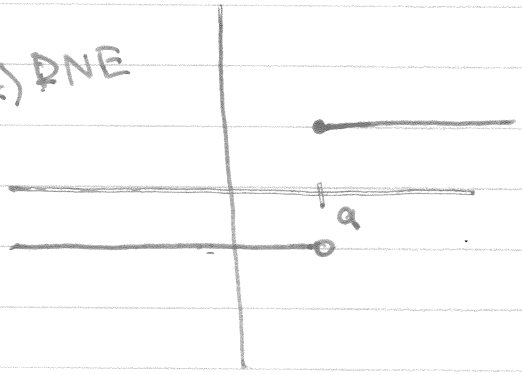
2. $f(x) = \begin{cases} -1 & x < a \\ 1 & x \geq a \end{cases}$

$\lim_{x \to a^+} f(x) = 1$

"approach from right"

$\lim_{x \to a^-} f(x) = -1$

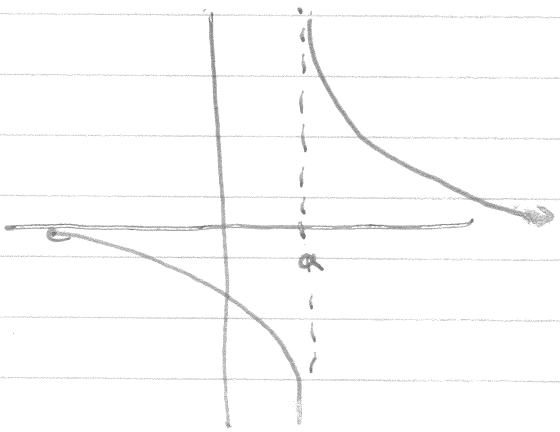
$\lim_{x \to a} f(x)$ DNE



Limits have to the same on both sides.

3. $f(x) = \dfrac{1}{x-a}$

$\lim_{x \to a} \dfrac{1}{x-a}$ = DNE



Def: A function $f(x)$ is "continuous at $x=a$" if

- $f(a)$ is defind
- $\lim_{x \to a} f(x)$ defined
- $\lim_{x \to a} f(x) = f(a)$

You need this to take the derivative of $f(x)$!

$x \to -\infty$

3, 9, 27, 81, 243, ...

DNE

DNE $\sqrt[3]{x}$

DNE

$\frac{-1}{x^3}$

6. $\lim_{x \to \infty} x^3$ $\lim_{x \to \infty} x^{1/3}$ $\lim_{x \to \infty} 3^x$ $\lim_{x \to \infty} x^{-3} = 0$